

Is the newly reported $X(5568)$ a $B\bar{K}$ molecular state?

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In this work, we perform a dynamical study of the $B^{(*)}$ and \bar{K} interaction and show that the newly reported $X(5568)$ or $X(5616)$ cannot be assigned to be an isovector $B\bar{K}$ or $B^*\bar{K}$ molecular state. We continue to investigate the isoscalar $B^{(*)}\bar{K}$ systems, and the $B^{(*)}\bar{K}$ systems with isospin $I = 0, 1$, and predict the existence of several isoscalar $B^{(*)}\bar{K}^{(*)}$ molecular states. A new task of exploring open-bottom molecular states will be created for future experiments.

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I. INTRODUCTION

In a recent experimental analysis [1], the DØ Collaboration reported a new enhancement structure $X(5568)$ in the $B_s^0\pi^\pm$ invariant mass spectrum, which has mass $m = 5567.8 \pm 2.9(\text{stat})_{-1.9}^{+0.9}(\text{syst})$ MeV and width $\Gamma = 21.9 \pm 6.4(\text{stat})_{-2.5}^{+5.0}(\text{syst})$ MeV [1]. Due to its observed decay mode, we conclude that the $X(5568)$ must contain four different valence quark components, which makes the $X(5568)$ a good candidate for a tetraquark state. Experimental and theoretical exploration of exotic multiquark states has become an intriguing issue, especially with the experimental progress on charmonium-like XYZ states and P_c pentaquark states in the past 12 years (see the review papers [2, 3] for more details).

Before presenting the detailed analysis, we first focus on the concrete experimental information released by DØ [1]. The DØ measurement shows that the $X(5568)$ has spin-parity quantum number $J^P = 0^+$. However, there exists the possibility that the mass of the enhancement structure appearing in the $B_s^0\pi^\pm$ invariant mass spectrum would be shifted by the addition of the nominal mass difference $m_{B_s^*} - m_{B_s}$ [1], which is due to the fact that the low-energy photons cannot be detected in the experiment. Thus, this enhancement structure may have a mass 5616 MeV, which corresponds to the $X(5616)$. Thus, the spin-parity of the $X(5616)$ is $J^P = 1^+$ [1].

Now that it has been observed $X(5568)$, theorists have paid more attention to the $X(5568)$. The popular explanation of the $X(5568)$ as a tetraquark state composed of a diquark and antidiquark was proposed in Refs. [4–9]. In this interpretation, the decay $X(5568) \rightarrow B_s^0\pi^\pm$ was calculated using the QCD sum rule approach [10–12], which supports the $X(5568)$ as a tetraquark state. By making a calibration by the mass of the $X(5568)$, its partner states were predicted in Ref. [13], where the color-magnetic interaction was adopted and the tetraquark scenario was considered. In Ref. [14], He and Ko analyzed the symmetry properties of the $X(5568)$ and its partners based on flavor SU(3) symmetry. Using a quark model with chromomagnetic interaction, the $X(5568)$ as a $su\bar{d}\bar{b}$ tetraquark

was studied in Ref. [15]. However, some groups hold opposite view. In a relativized quark model, the mass spectra of open-bottom tetraquark states were obtained [16]. They found that the $X(5568)$ disfavors the assignment of the $sq\bar{b}\bar{q}$ tetraquark state since the theoretical result is higher than the data. In Ref. [17], Esposito *et al.* calculated the mass of the $X_b = [\bar{b}\bar{q}]_{S=0}[sq']_{S=0}$ state using the constituent quark model, which has the same quantum number as that of $X(5568)$. The mass of the $X(5568)$ is below the obtained mass of X_b . Besides these tetraquark studies of the $X(5568)$, there were some discussions of the $X(5568)$ as the $B\bar{K}$ molecular state [18, 19]¹. In Ref. [18], the $B_s^0\pi^+$ decay width of the $X(5568)$ as the $B\bar{K}$ molecular state was estimated, which is comparable with the experimental data on $X(5568)$. A QCD sum rule study in Ref. [19] showed that a diquark-antidiquark configuration for the $X(5568)$ is more favorable than the $B\bar{K}$ molecular state picture.

In addition, the $X(5568)$ was explained to be the threshold effect [23]. We also noticed an investigation of the production of the $X(5568)$ in high-energy multiproduction process [24], where the authors indicated that it is hard to understand the large production rate of the $X(5568)$ using various general hadronization mechanisms. In recent work [25, 26], the difficulty of explaining the $X(5568)$ as the $B\bar{K}$ molecular state was indicated. The authors of Ref. [27] further found that the $X(5568)$ signal can be reproduced by using $B_s\pi - B\bar{K}$ coupled channel analysis, if the corresponding cutoff value is larger than a natural value $\Lambda \sim 1$ GeV. Thus, they concluded that it is difficult to explain the properties of the $X(5568)$. Later, a further study along this line was given in Ref. [28].

When facing different proposals for the $X(5568)$, a crucial task is to find the evidence to distinguish these different explanations from the $X(5568)$. In this work, we perform a serious dynamical study of the interaction between $B^{(*)}$ and \bar{K} using the one-boson exchange (OBE) model. In this investigation, we check whether $B^{(*)}$ and \bar{K} can be bound together to form a hadronic molecular state corresponding to the $X(5568)$ or the $X(5616)$.

This paper is organized as follows. We illustrate why the

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¹ There were some theoretical studies of the interactions between bottom-strange meson and kaon in Refs. [20–22].

$X(5568)$ or the $X(5616)$ cannot be a $\bar{B}^{(*)}K$ molecular state in Sec. II and Sec. III. In Sec. IV, we present the prediction of the possible $\bar{B}^{(*)}K^{(*)}$ molecular states. Finally, the paper ends with a short summary.

II. THE $X(5568)$ CANNOT BE AN S-WAVE $B\bar{K}$ MOLECULAR STATE

The quantum number $I(J^P)$ for the $X(5568)$ is constrained as $1(0^+)$, since it has the decay channel $B_s^0\pi^\pm$. The flavor wave functions $|I, I_3\rangle$ of the $B\bar{K}$ system are defined as $|1, 1\rangle = |B^+\bar{K}^0\rangle$, $|1, 0\rangle = \frac{1}{\sqrt{2}}(|B^+K^- \rangle - |B^0\bar{K}^0\rangle)$ and $|1, -1\rangle = |B^0K^- \rangle$. For the isoscalar $B\bar{K}$ system, its flavor wave function is $|0, 0\rangle = \frac{1}{\sqrt{2}}(|B^+K^- \rangle + |B^0\bar{K}^0\rangle)$. Here, we consider the S-wave $B\bar{K}$ molecular state [29–33], which has the same quantum number as that of the $X(5568)$. Thus, the spin-orbit wave function of the $B\bar{K}$ system corresponds to $|^1S_0\rangle$ with spin $S = 0$ and orbit $L = 0$. In fact, we notice that the mass of the $X(5568)$ is about 206 MeV lower than the $B\bar{K}$ threshold. This means that the $X(5568)$ should be a deeply bound state composed of B and \bar{K} if the $X(5568)$ is a $B\bar{K}$ molecular state. In the following, we need to carry out a quantitative dynamical calculation to test this scenario.

In the OBE model, the interaction between B and \bar{K} can be due to the light vector-meson (ρ and ω) exchanges. The corresponding effective Lagrangians describing the couplings of $B^{(*)}B^{(*)}\rho(\omega)$ [34, 35] and $\bar{K}^{(*)}\bar{K}^{(*)}\rho(\omega)$ [36] are

$$\mathcal{L}_{\bar{\rho}^{(*)}\bar{\rho}^{(*)}\Psi} = \sqrt{2}\beta g_V \bar{\rho}_a^\dagger \bar{\rho}_b \cdot \nabla_{ab} - \sqrt{2}\beta g_V \bar{\rho}_a^\dagger \cdot \bar{\rho}_b \cdot \nabla_{ab} - i2\sqrt{2}\lambda g_V \bar{\rho}_a^{\dagger\mu} \bar{\rho}_b^{*\nu} (\partial_\mu \nabla_\nu - \partial_\nu \nabla_\mu)_{ab}, \quad (1)$$

$$\begin{aligned} \mathcal{L}_{\rho\bar{K}^{(*)}\bar{K}^{(*)}} = & ig_{\rho\bar{K}\bar{K}} [\bar{K}^\dagger \vec{\tau} \cdot \partial^\mu \bar{K} \vec{\rho}_\mu - \partial^\mu \bar{K}^\dagger \vec{\tau} \cdot \bar{K} \vec{\rho}_\mu] \\ & + ig_{\rho\bar{K}^*\bar{K}^*} [(\partial^\mu \bar{K}^{*\nu\dagger} \bar{K}^* - \bar{K}^{*\dagger} \partial^\mu \bar{K}^{*\nu}) \vec{\tau} \cdot \vec{\rho}_\mu \\ & + (\bar{K}_\mu^{*\dagger} \partial^\mu \bar{K}^{*\nu} - \partial^\mu \bar{K}^{*\dagger} \bar{K}_\mu^{*\nu}) \vec{\tau} \cdot \vec{\rho}_\nu \\ & + (\bar{K}_\nu^{*\dagger} \bar{K}_\mu^* - \bar{K}_\mu^{*\dagger} \bar{K}_\nu^*) \vec{\tau} \cdot \partial^\mu \vec{\rho}^\nu], \end{aligned} \quad (2)$$

$$\begin{aligned} \mathcal{L}_{\omega\bar{K}^{(*)}\bar{K}^{(*)}} = & ig_{\omega\bar{K}\bar{K}} [\bar{K}^\dagger \partial^\mu \bar{K} \omega_\mu - \partial^\mu \bar{K}^\dagger \bar{K} \omega_\mu] \\ & + ig_{\omega\bar{K}^*\bar{K}^*} [(\partial^\mu \bar{K}^{*\nu\dagger} \bar{K}^* - \bar{K}^{*\dagger} \partial^\mu \bar{K}^{*\nu}) \omega_\mu \\ & + (\bar{K}_\mu^{*\dagger} \partial^\mu \bar{K}^{*\nu} - \bar{K}_\mu^{*\dagger} \bar{K}_\mu^{*\nu}) \omega_\nu \\ & + (\bar{K}_\nu^{*\dagger} \bar{K}_\mu^* - \bar{K}_\mu^{*\dagger} \bar{K}_\nu^*) \partial^\mu \omega^\nu], \end{aligned} \quad (3)$$

where the pseudoscalar $\bar{\rho}$ and vector $\bar{\rho}^*$ have the definition $\bar{\rho}^{(*)T} = (B^{(*)+}, B^{(*)0}, B_s^{(*)0})$. The vector matrix ∇ has the form

$$\nabla = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (4)$$

In addition, the coupling constants involved in Eq. (1) are taken as $\beta = 0.9$, $g_V = 5.8$, and $\lambda = 0.56 \text{ GeV}^{-1}$ [35], while the $KK\rho(\omega)$ constants $g_{\rho(\omega)K^{(*)}K^{(*)}}$ are

$$g_{\rho\bar{K}^{(*)}\bar{K}^{(*)}} = -\frac{1}{4}g_1 = -3.425,$$

$$g_{\omega\bar{K}^{(*)}\bar{K}^{(*)}} = -\frac{\sqrt{3}}{4}g_1 \cos\theta = -4.396,$$

which were given in Ref. [37].

The effective potential of the isovector $B\bar{K}$ system is deduced as

$$\mathcal{V}_{B\bar{K}}^{I=1}(r) = -\frac{\beta g_V}{2} [g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) - g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)] \quad (5)$$

In the above expression, the cutoff factor Λ denotes the phenomenological parameter around 1 GeV [29, 30], which is introduced in the monopole form factor $\mathcal{F}(q^2, m_E^2) = (\Lambda^2 - m_E^2)/(\Lambda^2 - q^2)$ when writing out the scattering amplitude of $B\bar{K} \rightarrow B\bar{K}$. Here, the function $Y(\Lambda, m, r)$ reads as

$$Y(\Lambda, m, r) = \frac{1}{4\pi r} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r}. \quad (6)$$

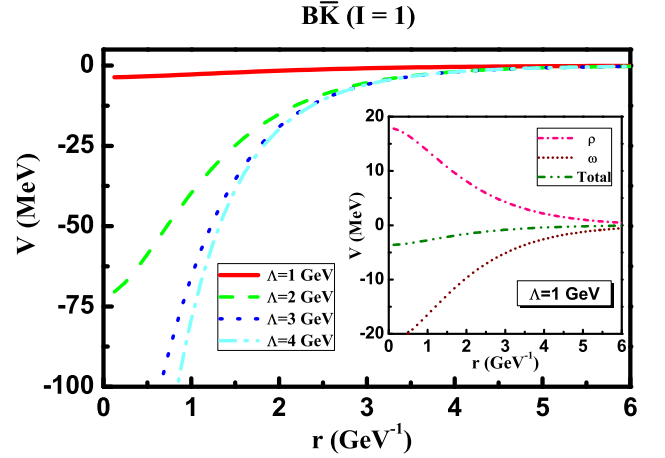


FIG. 1: The dependence of the OBE effective potential for the isovector S-wave $B\bar{K}$ system on r and typical Λ values. Here, we also show the variations of the subpotentials from the ρ and ω meson exchanges to r .

In Fig. 1, we first present the r dependence of effective potentials for the isovector $B\bar{K}$ system, where we take several typical values of the cutoff Λ . As showed in Fig. 1, the total OBE effective potentials corresponding to $\Lambda = 1 \sim 4 \text{ GeV}$ are attractive. As the values of Λ increases, the attraction between B and \bar{K} becomes stronger. Furthermore, we numerically solved the Schrödinger equation with the obtained effective potential, and could not find the corresponding bound-state solution for this S-wave isovector $B\bar{K}$ system when taking $\Lambda = 1 \sim 5 \text{ GeV}$ [29, 30], which means that the B and \bar{K} cannot be bound together to form an S-wave $B\bar{K}$ molecular state with isospin $I = 1$.

Since the $X(5568)$ was observed in the $B_s^+\pi^0$ channel, which is close to the mass of the $X(5568)$, we further consider the coupled-channel effect due to the mixing between the $B_s^+\pi^0$ and $B^+\bar{K}^0$ channels. In our calculation, we adopt the effective potential [36]

$$\mathcal{L}_{\pi\bar{K}\bar{K}^*} = ig_{\pi\bar{K}\bar{K}^*} [\bar{K}^\dagger \vec{\tau} \cdot \bar{K}^{*\mu} \partial_\mu \vec{\pi} - \bar{K}^\dagger \vec{\tau} \cdot \partial_\mu \bar{K}^{*\mu} \vec{\pi}] + H.c., \quad (7)$$

where $g_{\pi\bar{K}\bar{K}^*} = \frac{1}{4}g_1$ [37]. Then, the obtained total effective potentials corresponding to the discussed $X(5568)$ can be written as

$$\mathcal{V}(r) = \begin{pmatrix} \langle B_s\pi|V|B_s\pi\rangle & \langle B_s\pi|V|B\bar{K}\rangle \\ \langle B\bar{K}|V|B_s\pi\rangle & \langle B\bar{K}|V|B\bar{K}\rangle \end{pmatrix} \quad (8)$$

with

$$\begin{aligned} \langle B_s\pi|V|B_s\pi\rangle &= 0, \\ \langle B_s\pi|V|B\bar{K}\rangle &= \langle B\bar{K}|V|B_s\pi\rangle \\ &= \frac{\sqrt{2}}{4}\beta g_V g_{\pi\bar{K}\bar{K}^*}(m_\pi + m_K) Y(\Lambda, m_{K^*}, r), \\ \langle B\bar{K}|V|B\bar{K}\rangle &= -\frac{\beta g_V}{2} [g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) - g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)]. \end{aligned}$$

With this deduced effective potential, we solve the coupled-channel Schrödinger equation. Unfortunately, we still cannot find the bound-state solutions when scanning the range $\Lambda = 1 \sim 5$ GeV.

According to our study, we can fully exclude the $X(5568)$ as an isovector S-wave $B\bar{K}$ molecular state with $J^P = 0^+$, which is consistent with the conclusion made in Refs. [38, 39].

III. THE $X(5616)$ CANNOT BE AN S-WAVE $B^*\bar{K}$ MOLECULAR STATE

Since the quantum number $I(J^P)$ of the $X(5616)$ is $1(1^+)$ [1], the S-wave $B^*\bar{K}$ molecular state is possible assignment for the $X(5616)$. If we only consider the S-wave interaction between B^* and \bar{K} mesons, the obtained OBE effective potential is

$$\mathcal{V}_{B^*\bar{K}}^{I=1}(r) = -\frac{\beta g_V}{2} [g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) - g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)], \quad (9)$$

which is the same as the expression in Eq. (5). The difference between $B\bar{K}$ and $B^*\bar{K}$ with $I = 1$ can be seen in the difference of their reduced masses. Although the total effective potential of an S-wave $B^*\bar{K}$ system with isospin $I = 1$ is attractive, we cannot find the corresponding bound-state solution.

When further considering the S-D mixing effect on the $B^*\bar{K}$ system since there exists mixing of the $B^*\bar{K}$ systems with spin-orbit wave functions $|^3S_1\rangle$ and $|^3D_1\rangle$, the effective potential in Eq. (9) should be modified as

$$\begin{aligned} \mathcal{V}_{B^*\bar{K}}^{I=1}(r) &= -\frac{\beta g_V}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} [g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) \\ &\quad - g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)], \end{aligned} \quad (10)$$

which is a 2×2 matrix, where the matrix $\text{diag}(1, 1)$ is deduced from

$$\begin{pmatrix} \langle ^3S_1 | \epsilon_1 \cdot \epsilon_3^\dagger | ^3S_1 \rangle & \langle ^3S_1 | \epsilon_1 \cdot \epsilon_3^\dagger | ^3D_1 \rangle \\ \langle ^3D_1 | \epsilon_1 \cdot \epsilon_3^\dagger | ^3S_1 \rangle & \langle ^3D_1 | \epsilon_1 \cdot \epsilon_3^\dagger | ^3D_1 \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (11)$$

Here, ϵ_1 and ϵ_3^\dagger correspond to the operators of the polarization vectors of the initial and final B^* meson, respectively.

To search for the bound-state solution, we solve the coupled-channel Schrödinger equation with Eq. (10). The bound-state solution is still absent when we scan the range $\Lambda = 1 \sim 5$ GeV in our numerical analysis.

In our calculation, we further consider the coupled-channel effect with the $B_s^*\pi$ and $B^*\bar{K}$ channels. However, the bound solutions cannot be obtained.

Thus, our study does not support the $X(5616)$ as an isovector S-wave $B^*\bar{K}$ molecular state.

IV. THE PREDICTION OF POSSIBLE $B^{(*)}\bar{K}^{(*)}$ MOLECULAR STATES

A. Isoscalar $B\bar{K}$ and $B^*\bar{K}$ systems

In the above sections, we discussed isovector $B\bar{K}$ and $B^*\bar{K}$ systems, which also stimulates our interest in further studying other $B^{(*)}\bar{K}^{(*)}$ systems. First, we focus on the isoscalar $B\bar{K}$ and $B^*\bar{K}$ systems. Their OBE effective potentials are

$$\begin{aligned} \mathcal{V}_{B\bar{K}}^{I=0}(r) &= \frac{\beta g_V}{2} [3g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) + g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)] \quad (12) \\ \mathcal{V}_{B^*\bar{K}}^{I=0}(r) &= \frac{\beta g_V}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} [3g_{\rho\bar{K}\bar{K}} Y(\Lambda, m_\rho, r) \\ &\quad + g_{\omega\bar{K}\bar{K}} Y(\Lambda, m_\omega, r)]. \end{aligned} \quad (13)$$

When comparing the OBE effective potentials of the isoscalar and isovector $B^{(*)}\bar{K}$ systems, we find that an isospin factor -3 is introduced in the ρ -exchange potentials for these isoscalar systems, while the isoscalar and isovector $B^{(*)}\bar{K}$ systems have the same ω -exchange potential. The behaviors of the effective potentials of the isoscalar $B^{(*)}\bar{K}$ systems make that it easier to form the isoscalar $B^{(*)}\bar{K}$ molecular states. By solving the Schrödinger equation, we confirm the above speculation, namely that we can find the bound-state solutions for the isoscalar $B^{(*)}\bar{K}$ systems. In Table. I, we list the obtained binding energy, root-mean-square radius and the corresponding Λ values. When taking $\Lambda = 1.9$ GeV, there exist shallow isoscalar $B^{(*)}\bar{K}$ molecular states. As the value of Λ increases, the binding energies of these two systems become deeper. Here, the input of Λ is not far away from 1 GeV, which come from studying the nuclear force [29, 30]. Thus, we may conclude that there probably exist isoscalar $B\bar{K}$ and $B^*\bar{K}$ molecular states, which have the quantum numbers $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$, respectively.

In fact, the above formula can be extended to the discussion of the DK system with $(I = 0, J = 0)$ and the D^*K system with $(I = 0, J = 1)$. Our calculation shows that the masses of the $D_{s0}(2317)$ and the $D_{s1}^*(2460)$ [40] can be reproduced when the cutoff Λ is taken around 3.5 GeV, where the $D_{s0}(2317)$ and the $D_{s1}^*(2460)$ correspond to the DK system with $(I = 0, J = 0)$ and the D^*K system with $(I = 0, J = 1)$, respectively, since the reduced masses of the $B\bar{K}$ and $B^*\bar{K}$ systems are heavier than those of the DK and D^*K systems, respectively. Thus, we can conclude that the cutoff Λ for $B\bar{K}/B^*\bar{K}$ should be smaller than that of DK/D^*K . The numerical results listed in Table I indeed can reflect this point.

TABLE I: The Λ dependence of the obtained bound-state solutions (binding energy E and root-mean-square radius r_{RMS}) for isoscalar $B^{(*)}\bar{K}$ systems. Here, E , r_{RMS} , and Λ are in units of MeV, fm, and GeV, respectively.

State	Λ	E	r_{RMS}	State	Λ	E	r_{RMS}
$[B\bar{K}]_{J=0}^{I=0}$	1.90	-0.29	5.66	$[B^*\bar{K}]_{J=1}^{I=0}$	1.90	-0.30	5.64
	2.10	-4.36	2.45		2.10	-4.40	2.44
	2.30	-11.69	1.58		2.30	-11.76	1.57

If isoscalar $B\bar{K}$ and $B^*\bar{K}$ molecular states exist, finding them becomes a crucial task. For an isoscalar $B\bar{K}$ molecular state, its two-body and three-body Okubo-Zweig-Iizuka-allowed decay channels are forbidden. Thus, experimental searches for this isoscalar $B\bar{K}$ are very difficult. For an isoscalar $B^*\bar{K}$ molecular state, we suggest an experiment to further analyze its $B_s\pi\pi$ final state, by which this isoscalar $B^*\bar{K}$ molecular state can be discovered.

B. The $B\bar{K}^*$ and $B^*\bar{K}^*$ systems

Besides the systems discussed in Sec. II and IV A, in this work we also investigate the $B\bar{K}^*$ and $B^*\bar{K}^*$ systems. For the $B^*\bar{K}^*$ systems, there also exist π and η meson-exchange contributions to the effective potentials. In deducing the effective potentials, we need to adopt the following effective Lagrangians:

$$\mathcal{L}_{\bar{\Phi}^*\Phi^*\mathbb{P}} = i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}V^\alpha\bar{\Phi}_a^{*\mu\dagger}\bar{\Phi}_b^{*\lambda}\partial^\nu\mathbb{P}_{ab}, \quad (14)$$

$$\mathcal{L}_{\pi\bar{K}^*\bar{K}^*} = -g_{\pi\bar{K}^*\bar{K}^*}\varepsilon^{\mu\nu\rho\sigma}\partial_\rho\bar{K}_\sigma^{*\dagger}\bar{\tau}\cdot\partial_\mu\bar{K}_\nu^*\vec{\pi}, \quad (15)$$

$$\mathcal{L}_{\eta\bar{K}^*\bar{K}^*} = g_{\eta\bar{K}^*\bar{K}^*}\varepsilon^{\mu\nu\rho\sigma}\partial_\rho\bar{K}_\sigma^{*\dagger}\partial_\mu\bar{K}_\nu^*\eta \quad (16)$$

with

$$\mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (17)$$

Here, $g = 0.59$ is extracted from the experimental width of D^{*+} [41], and the pion decay constant $f_\pi = 132$ MeV. Additionally, $g_{\pi\bar{K}^*\bar{K}^*}$ and $g_{\eta\bar{K}^*\bar{K}^*}$ are expressed by $g_{\pi\bar{K}^*\bar{K}^*} = \frac{g_1^2 N_c}{64\pi^2 f_\pi}$, and $g_{\eta\bar{K}^*\bar{K}^*} = \frac{g_1^2 N_c}{64\sqrt{3}\pi^2 f_\pi}$ [42] with the number of colors N_c , where the value of g_1 was given in Sec. II.

Here, the S-D mixing effect is also taken into account, and the relevant spin-orbit wave functions $|^{2S+1}L_J\rangle$ include

$$\begin{aligned} B\bar{K}^* : & |^3S_1\rangle, |^3D_1\rangle, \\ B^*\bar{K}^* : & |^1S_0\rangle, |^5D_0\rangle, \\ & |^3S_1\rangle, |^3D_1\rangle, |^5D_1\rangle, \\ & |^5S_2\rangle, |^1D_2\rangle, |^3D_2\rangle, |^5D_2\rangle. \end{aligned} \quad (18)$$

The obtained general expressions of the $B\bar{K}^*$ and $B^*\bar{K}^*$ systems when considering the S-D mixing effect read

$$\begin{aligned} \mathcal{V}_{B\bar{K}^*}^I(r) = & \frac{1}{2}\mathcal{G}(I)\beta g_V g_{\rho\bar{K}^*\bar{K}^*} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} Y(\Lambda, m_\rho, r) \\ & + \frac{1}{2}\beta g_V g_{\omega\bar{K}^*\bar{K}^*} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} Y(\Lambda, m_\omega, r), \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{V}_{B^*\bar{K}^*}^{I,J}(r) = & \frac{1}{6\sqrt{2}}\frac{gg_{\pi\bar{K}^*\bar{K}^*}}{f_\pi}\mathcal{G}(I)\left[\mathcal{E}_1(J)\nabla^2 + \mathcal{S}(J)r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right] \\ & \times Y(\Lambda, m_\pi, r) - \frac{1}{6\sqrt{6}}\frac{gg_{\eta\bar{K}^*\bar{K}^*}}{f_\pi}\left[\mathcal{E}_1(J)\nabla^2 \right. \\ & \left. + \mathcal{S}(J)r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right]Y(\Lambda, m_\eta, r) \\ & - \frac{1}{2}\beta g_V g_{\rho\bar{K}^*\bar{K}^*}\mathcal{G}(I)\mathcal{E}_2(J)Y(\Lambda, m_\rho, r), \\ & + \frac{1}{2}\beta g_V g_{\omega\bar{K}^*\bar{K}^*}\mathcal{E}_2(J)Y(\Lambda, m_\omega, r), \end{aligned} \quad (20)$$

where the superscripts I and J denote the isospin and total angular momentum of these discussed systems. $\mathcal{G}(I)$ is the isospin factor, which is taken as -3 for the isoscalar system, and 1 for the isovector system. The concrete forms of $\mathcal{E}_1(J)$, $\mathcal{E}_2(J)$, and $\mathcal{S}(J)$ are $\mathcal{E}_1(0) = \text{diag}(2, -1)$, $\mathcal{E}_1(1) = \text{diag}(1, 1, -1)$, $\mathcal{E}_1(2) = \text{diag}(-1, 2, 1, -1)$, $\mathcal{E}_2(0) = \text{diag}(1, 1)$, $\mathcal{E}_2(1) = \text{diag}(1, 1, 1)$, $\mathcal{E}_2(2) = \text{diag}(1, 1, 1, 1)$,

$$\begin{aligned} \mathcal{S}(0) = & \begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}, \mathcal{S}(1) = \begin{pmatrix} 0 & -\sqrt{2} & 0 \\ -\sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ and } \mathcal{S}(2) = \\ & \begin{pmatrix} 0 & \sqrt{3} & 0 & -\sqrt{\frac{14}{5}} \\ \sqrt{3} & 0 & 0 & -\frac{2}{\sqrt{7}} \\ 0 & 0 & -1 & 0 \\ -\sqrt{\frac{14}{5}} & -\frac{2}{\sqrt{7}} & 0 & -\frac{3}{7} \end{pmatrix}. \end{aligned}$$

With the above preparation, we try to search for the bound solutions by solving the Schrödinger equation. In Table II, the obtained results are collected. Among the discussed isovector $B\bar{K}^*$ and $B^*\bar{K}^*$ systems, only the $B^*\bar{K}^*$ system with $J = 0$ has a bound-state solution when Λ is around 3 GeV, which is obviously different from 1 GeV [29, 30]. Thus, if strictly considering this criterion of the Λ value, we conclude that there do not exist isovector $B^{(*)}\bar{K}^*$ molecular states. Different from the isovector case, the isoscalar $B^{(*)}\bar{K}$ systems may exist, as shown in Table II. In the following, we further discuss their allowed decay modes:

1. The $B\bar{K}^*$ molecular state with $(I = 0, J = 1)$ can decay into $B^*\bar{K}$, $B_s\omega$ and $B_s^*\eta$.
2. $B_s^*\omega$ is an allowed decay mode of the $B^*\bar{K}^*$ molecular state with $(I = 0, J = 2)$.
3. The allowed decay channels of the $B^*\bar{K}^*$ molecular state with $(I = 0, J = 1)$ include $B^*\bar{K}$, $B\bar{K}^*$, $B_s\omega$, and $B_s^*\omega$.
4. $B\bar{K}$, $B_s\eta$ and $B_s^*\omega$ are the allowed two-body decay channels for the $B^*\bar{K}^*$ state with $(I = 0, J = 0)$.

TABLE II: The Λ dependence of the obtained bound-state solutions (binding energy E and root-mean-square radius r_{RMS}) of the $B\bar{K}^*$ and $B^*\bar{K}^*$ systems. Here, E , r_{RMS} , and Λ are in units of MeV, fm, and GeV, respectively.

State	Λ	E	r_{RMS}	State	Λ	E	r_{RMS}
$[B\bar{K}^*]_{J=1}^{I=0}$	1.40	-0.32	5.16	$[B\bar{K}^*]_{J=1}^{I=1}$
	1.60	-10.30	1.37	
	1.80	-30.20	0.88	
$[B^*\bar{K}^*]_{J=0}^{I=0}$	0.88	-0.60	4.91	$[B^*\bar{K}^*]_{J=0}^{I=1}$	3.00	-0.98	3.67
	1.08	-6.06	2.04		3.30	-6.57	1.55
	1.28	-20.97	1.24		3.60	-19.34	0.94
$[B^*\bar{K}^*]_{J=1}^{I=0}$	1.60	-1.15	3.62	$[B^*\bar{K}^*]_{J=1}^{I=1}$
	1.80	-8.69	1.54	
	2.00	-22.40	1.06	
$[B^*\bar{K}^*]_{J=2}^{I=0}$	1.10	-0.14	5.77	$[B^*\bar{K}^*]_{J=2}^{I=1}$
	1.20	-7.41	1.57	
	1.30	-24.48	0.97	

In our calculation, we also extend our study to the charm sector. The relevant numerical results for the DK^* and D^*K^* systems are collected in Table III.

TABLE III: The Λ dependence of the obtained bound-state solutions (binding energy E and root-mean-square radius r_{RMS}) of the DK^* and D^*K^* systems. Here, E , r_{RMS} , and Λ are in units of MeV, fm, and GeV, respectively.

State	Λ	E	r_{RMS}	State	Λ	E	r_{RMS}
$[DK^*]_{J=1}^{I=0}$	1.60	-0.90	4.18	$[DK^*]_{J=1}^{I=1}$
	1.80	-9.30	1.56	
	2.00	-23.87	1.05	
$[D^*K^*]_{J=0}^{I=0}$	1.00	-0.79	4.76	$[D^*K^*]_{J=0}^{I=1}$	3.70	-0.46	4.92
	1.20	-6.97	2.05		4.10	-7.88	1.56
	1.40	-22.51	1.27		4.50	-28.87	0.85
$[D^*K^*]_{J=1}^{I=0}$	1.80	-0.89	4.25	$[D^*K^*]_{J=1}^{I=1}$
	2.20	-15.92	1.29	
	2.60	-47.17	0.84	
$[D^*K^*]_{J=2}^{I=0}$	1.20	-0.21	5.66	$[D^*K^*]_{J=2}^{I=1}$
	1.30	-6.80	1.77	
	1.40	-21.52	1.09	

These numerical results shown in Table III indicate that the isoscalar DK^* and D^*K^* states are very promising molecular candidates. Their decay behaviors are

$$[DK^*]_{J=1}^{I=0} \rightarrow D^*K, D_s\eta, D_s^*\omega,$$

$$\begin{aligned} [D^*K^*]_{J=1}^{I=0} &\rightarrow DK, D_s\eta, D_s^*\omega, \\ [D^*K^*]_{J=1}^{I=0} &\rightarrow D^*K, DK^*, D_s\omega, D_s^*\omega, \\ [D^*K^*]_{J=2}^{I=0} &\rightarrow D_s^*\omega. \end{aligned}$$

It is obvious that experimental searches for these predicted isoscalar $B^{(*)}\bar{K}^*$ and $D^{(*)}K^*$ molecular states will be an intriguing issue. The above information is valuable to further study them experimentally.

V. SUMMARY

Stimulated by the recent evidence of a new enhancement structure $X(5568)$ or $X(5616)$ [1], we carried out a study of the interactions of isovector $B\bar{K}$ and $B^*\bar{K}$ systems via the OBE model. This dynamical study makes us exclude the $X(5568)$ or the $X(5616)$ as the isovector $B\bar{K}$ or $B^*\bar{K}$ molecular state. In Refs. [25, 26], the difficulty of assigning the $X(5568)$ to be the $B\bar{K}$ molecular state was discussed. Obviously, we reach the same conclusion using different approaches.

In this work, we also studied isoscalar $B\bar{K}$ and $B^*\bar{K}$ systems; we predicted that there isoscalar $B\bar{K}$ and $B^*\bar{K}$ molecular states may exist, and their decay behaviors were discussed. In addition, we also focused on the $B^{(*)}\bar{K}^*$ systems. Our calculation illustrates that $B^{(*)}$ and \bar{K}^* cannot form isovector molecular states, but they can be bound together to construct isoscalar $B^{(*)}\bar{K}^*$ molecular states. The allowed decay modes of these possible isoscalar $B^{(*)}\bar{K}^*$ molecular states show that it is possible to find them in experiments. Thus, we suggest future experimental exploration of these isoscalar open-bottom molecular states.

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Note added—When preparing the manuscript, we noticed the preliminary result from the LHCb experiment [43], where the signal of $X(5568)$ was not observed. In Ref. [43], the LHCb's analysis also shows that the cone cut selection criterion can generate broad peaking structures. The DØ Collaboration performed an analysis of the $B_s^0\pi^+$ data with and without the cone cut, which indicates that there exists a structure with and without the cone cut. Here, the cone cut clearly enhances the resonance state as analyzed in Ref. [1]. According to our present study, we can deny the possibility of the $X(5568)$ or $X(5616)$ as an isoscalar $B\bar{K}$ or B^*K hadronic molecular state.

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